Dilations Investigation—Student Activity

**Title:** Dilation Investigation

**Subject:** Coordinate transformations in geometry (Chapter 8)

**Objective:** Given grid paper, a centimeter ruler, a protractor, and a sheet of patty paper the students will generate and apply the relationship between the scale factor of dilation and the coordinates of the image, angle measures, perimeter, and area by completing data tables and by redrawing figures on graph paper.

**TEKS Focus:** 8.6A, B The student is expected to generate similar figures using dilations including enlargements and reductions and graph dilations, reflections, and translations on a coordinate plane.

G.11A The student is expected to use and extend similarity properties and transformations to explore and justify conjectures about geometric figures.

**Additional TEKS covered:** 8.10A The student is expected to describe the resulting effects on perimeter and area when dimensions of a shape are changed proportionally.

G.11D The student is expected to describe the effect on perimeter, area, (and volume) when one or more dimensions of a figure are changed and apply this idea in solving problems.

8.7D The student is expected to locate and name points on a coordinate plane using ordered pairs of rational numbers.

**Lesson Overview:** In part one, students will use patty paper to create a dilation with a scale factor of two on plane paper. Students will then transfer the dilation to the coordinate grid and create additional dilations of the same figure with scale factors of three and one-half.

In part two, students will make coordinate dilations of several figures and will look at the effect of the scale change on the coordinates, side length, perimeter, and area of each image.

**Extension:** Students can graph the coordinates of the four triangles in part one using a graphing calculator and determine the equations for the rays used in the dilation.

**Grouping:** Students can work individually or in groups.

**Materials:** Each student will need a copy of the lesson, a separate sheet of one centimeter grid paper, a centimeter ruler, a protractor, and a sheet of patty paper.

**Procedure:** Students can use the same piece of patty paper for steps one through eight to transfer the dilations to the grid paper. Students should be able to work through the document on their own with teacher guidance.

**Homework:** Students can complete part two of the assignment as homework. Most students should be able to complete both parts in a ninety minute class period.
Dilations Investigation—Student Activity

Name: Answer Key _________________________________ Geometry

Dilations – Part I

1. On the picture below, draw rays from \( O \) through each vertex of \( \triangle ABC \).

2. Use patty paper to measure the length of \( OA \). Place \( A' \) on \( OA \) so that \( OA' = 2 \cdot OA \).

3. Use the same process to find \( B' \) and \( C' \).

4. Draw \( \triangle A'B'C' \).

5. Trace \( \triangle ABC \) on a piece of patty paper. Compare the angles of \( \triangle ABC \) with the angles of \( \triangle A'B'C' \). What do you notice about the angle measures?

6. Use a ruler to find the following lengths in centimeters.
   - \( AB = 3 \text{ cm} \)
   - \( BC = 4 \text{ cm} \)
   - \( AC = 5 \text{ cm} \)
   - \( A'B' = 6 \text{ cm} \)
   - \( B'C' = 8 \text{ cm} \)
   - \( A'C' = 10 \text{ cm} \)

7. What do you notice about the lengths of the sides of the two triangles?

   The lengths of \( \triangle A'B'C' \) are twice as long as the lengths of \( \triangle ABC \).

8. The Scale Factor for this dilation is \( 2 \) or \( 2:1 \).
Dilations Investigation—Student Activity

9. Use your patty paper to transfer $\triangle ABC$ and $\triangle A'B'C'$ to the grid paper with point $O$ at the origin. Write the ordered pairs next to each vertex on the grid paper. What do you notice about these ordered pairs?

*The coordinates of $\triangle A'B'C'$ are twice as big as the coordinates of $\triangle ABC$*

10. Dilate $\triangle ABC$ using a scale factor of 3 on your grid paper. Label the new triangle $\triangle A''B''C''$. Write the ordered pairs next to each vertex.

11. Dilate $\triangle ABC$ using a scale factor of $\frac{1}{2}$ on your grid paper. Label the triangle $\triangle A'''B'''C'''$. Write the ordered pairs next to each vertex.

<table>
<thead>
<tr>
<th>Original Triangle</th>
<th>Dilation (Scale Factor of 2)</th>
<th>Dilation (Scale Factor of 3)</th>
<th>Dilation (Scale Factor of $\frac{1}{2}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>$(3, 5)$</td>
<td>$A'$</td>
<td>$(6, 10)$</td>
</tr>
<tr>
<td>$B$</td>
<td>$(3, 2)$</td>
<td>$B'$</td>
<td>$(6, 4)$</td>
</tr>
<tr>
<td>$C$</td>
<td>$(7, 2)$</td>
<td>$C'$</td>
<td>$(14, 4)$</td>
</tr>
</tbody>
</table>

12. What observations/conjectures can you make about dilations based on your work so far?

*Possible answers:*

*When the scale factor is two, you multiply the coordinates by 2. When the scale factor is three, you multiply the coordinates by three. When the scale factor is $\frac{1}{2}$, you multiply the coordinates by $\frac{1}{2}$.***

13. Complete this conjecture based on Table 1: The image of $(a, b)$ after a dilation with center $O$ and scale factor $k$ has coordinates $(ka, kb)$. 

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Dilations – Part II

1. Record your observations about the characteristics of the triangles $\triangle ABC$ and $\triangle A'B'C'$ in the table below.

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Original $\triangle ABC$</th>
<th>$\triangle A'B'C'$ (Scale Factor of 2)</th>
<th>Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coordinates</td>
<td>A(3, 5) B(3, 2) C(7, 2)</td>
<td>A(6, 10) B(6, 4) C(14, 4)</td>
<td>Doubled</td>
</tr>
<tr>
<td>Angle Measures</td>
<td>$53^\circ - 90^\circ - 37^\circ$</td>
<td>$53^\circ - 90^\circ - 37^\circ$</td>
<td>Same measure</td>
</tr>
<tr>
<td>Length of Sides</td>
<td>3 cm–4 cm–5 cm</td>
<td>6 cm–8 cm–10 cm</td>
<td>Doubled</td>
</tr>
<tr>
<td>Perimeter</td>
<td>12 cm</td>
<td>24 cm</td>
<td>Doubled</td>
</tr>
<tr>
<td>Area</td>
<td>6 cm$^2$</td>
<td>24 cm$^2$</td>
<td>Quadrupled</td>
</tr>
</tbody>
</table>

2. Graph Quadrilateral $JKLM$ on the back of your grid paper. Dilate quadrilateral $JKLM$ with a scale factor of 3 on your grid paper. Label the image $J'K'L'M'$ and complete the table below.

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Original $JKLM$</th>
<th>Quad. $JKLM'$ (Scale Factor of 3)</th>
<th>Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coordinates</td>
<td>J(-2, 1) K(-2, 3), L(3, 3), M(3, 1)</td>
<td>J'(-6, 3) K'(-6, 9), L'(9, 9), M'(9, 3)</td>
<td>Tripled</td>
</tr>
<tr>
<td>Angle Measures</td>
<td>$90^\circ - 90^\circ - 90^\circ - 90^\circ$</td>
<td>$90^\circ - 90^\circ - 90^\circ - 90^\circ$</td>
<td>Same measure</td>
</tr>
<tr>
<td>Length of Sides</td>
<td>2 cm by 5 cm</td>
<td>6 cm by 15 cm</td>
<td>Tripled</td>
</tr>
<tr>
<td>Perimeter</td>
<td>14 cm</td>
<td>42 cm</td>
<td>Tripled</td>
</tr>
<tr>
<td>Area</td>
<td>10 cm$^2$</td>
<td>90 cm$^2$</td>
<td>Multiplied by 9</td>
</tr>
</tbody>
</table>

3. Graph Quadrilateral $QRST$ on the same grid paper. Dilate quadrilateral $QRST$ with a scale factor of $\frac{1}{2}$ on your grid paper. Label the image $QRST'$ and complete the table below.

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Original $QRST$</th>
<th>Quad. $QRST'$ (Scale Factor of $\frac{1}{2}$)</th>
<th>Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coordinates</td>
<td>Q(-2,-2) R(-2, -4), S(-6, -4), T(-6, 2)</td>
<td>Q'(-1, -1) R'(-1, -2), S'(-3, -2), T'(-3, 1)</td>
<td>One half</td>
</tr>
<tr>
<td>Angle Measures</td>
<td>$135^\circ - 90^\circ - 90^\circ - 45^\circ$</td>
<td>$135^\circ - 90^\circ - 90^\circ - 45^\circ$</td>
<td>Same measure</td>
</tr>
<tr>
<td>Length of Sides</td>
<td>2 cm–4 cm–6 cm–5.7 cm</td>
<td>1 cm–2 cm–3 cm–2.9 cm</td>
<td>One half</td>
</tr>
<tr>
<td>Perimeter</td>
<td>17.7 cm</td>
<td>8.8 cm</td>
<td>One half</td>
</tr>
<tr>
<td>Area</td>
<td>16 cm$^2$</td>
<td>4 cm$^2$</td>
<td>One fourth</td>
</tr>
</tbody>
</table>
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4. A diameter of a circle has endpoints A and B. Graph the circle on your grid paper. Dilate the circle with a scale factor of 2. Label the coordinates of the image \( A' \) and \( B' \) and complete the table below.

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Original circle with diameter ( AB )</th>
<th>New circle with diameter ( A'B' )</th>
<th>Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coordinates</td>
<td>( A(4,1), B(4,-3) )</td>
<td>( A'(8,2), B'(8,-6) )</td>
<td>Doubled</td>
</tr>
<tr>
<td>Length of the diameter</td>
<td>( 4 \text{ cm} )</td>
<td>( 8 \text{ cm} )</td>
<td>Doubled</td>
</tr>
<tr>
<td>Length of the radius</td>
<td>( 2 \text{ cm} )</td>
<td>( 4 \text{ cm} )</td>
<td>Doubled</td>
</tr>
<tr>
<td>Circumference</td>
<td>( 4\pi \approx 12.56 \text{ cm} )</td>
<td>( 8\pi \approx 25.12 \text{ cm} )</td>
<td>Doubled</td>
</tr>
<tr>
<td>Area</td>
<td>( 4\pi \approx 12.56 \text{ cm}^2 )</td>
<td>( 16\pi \approx 50.24 \text{ cm}^2 )</td>
<td>Quadrupled</td>
</tr>
</tbody>
</table>

5. Based on the information in all four tables, what conjectures can you make about:
   a. Scale Factor used in the dilation and angle measures. *Scale factor has no effect on angle measure*
   b. Scale Factor used in the dilation and side lengths/diameter. *Side length is multiplied by the scale factor*
   c. Scale Factor used in the dilation and perimeter/circumference. *Perimeter is multiplied by the scale factor*
   d. Scale Factor used in the dilation and area. *Area is multiplied by the scale factor squared.*

6. Use the graph to below to answer the following dilation questions.

   a) Find the coordinates of triangle \( A'B'C' \) after a dilation with a scale factor of 3.
      \[ A'(-9,6) B'(9,-6) C'(9,12) \]
      How does this change angle measures?
      *Angles are the same*

   b) Find the coordinates of triangle \( A'B'C' \) after a dilation with a scale factor of \( \frac{3}{2} \).
      \[ A'(-4.5,3) B'(4.5,-3) C'(4.5,6) \]
      How does this change the lengths of the sides?
      *Side length is 3/2 bigger*

   c) If the coordinates of \( A' \) are \((-1.5, 1) \) after a dilation, find the coordinates of \( B' \) and \( C' \)?
      \[ B'(1.5,-1) C'(1.5,2) \]
      How does this change the perimeter of the triangle?
      *The perimeter is half as big*

   d) If the coordinates of \( B' \) are \((18, -12) \) after a dilation, find the coordinates of \( A' \) and \( C' \)?
      \[ A'(-12,8) C'(12,16) \]
      How does this change the area of the triangle?
      *The area is \( 6^2 \) or 36 times bigger*
Dilations Investigation—Teacher resource

Name: ___________________________      Geometry
Dilations Investigation—Student Activity

Name: ____________________________________________

Dilations – Part I

1. On the picture below, draw rays from \( O \) through each vertex of \( \triangle ABC \).

2. Use patty paper to measure the length of \( \overline{OA} \). Place \( A' \) on \( \overline{OA} \) so that \( OA' = 2 \cdot OA \).

3. Use the same process to find \( B' \) and \( C' \).

4. Draw \( \triangle A'B'C' \).

5. Trace \( \triangle ABC \) on a piece of patty paper. Compare the angles of \( \triangle ABC \) with the angles of \( \triangle A'B'C' \). What do you notice about the angle measures?

6. Use a ruler to find the following lengths in centimeters.

   a. \( AB = \) ______________
   b. \( BC = \) ______________
   c. \( AC = \) ______________
   d. \( A'B' = \) ______________
   e. \( B'C' = \) ______________
   f. \( A'C' = \) ______________

7. What do you notice about the lengths of the sides of the two triangles?

8. The **Scale Factor** for this dilation is _______.

![Diagram of \( \triangle ABC \) with rays from \( O \) through each vertex]
Dilations Investigation—Student Activity

9. Use your patty paper to transfer \( \triangle ABC \) and \( \triangle A'B'C' \) to the grid paper with point \( O \) at the origin. Write the ordered pairs next to each vertex on the grid paper and in Table 1 below. What do you notice about these ordered pairs?

10. Dilate \( \triangle ABC \) using a scale factor of 3 on your grid paper. Label the new triangle \( \triangle A''B''C'' \). Write the ordered pairs next to each vertex and in Table 1 below.

11. Dilate \( \triangle ABC \) using a scale factor of \( \frac{1}{2} \) on your grid paper. Label the triangle \( \triangle A'''B'''C''' \). Write the ordered pairs next to each vertex and in Table 1 below.

<table>
<thead>
<tr>
<th>Original Triangle</th>
<th>Dilation (Scale Factor of 2)</th>
<th>Dilation (Scale Factor of 3)</th>
<th>Dilation (Scale Factor of ( \frac{1}{2} ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A )</td>
<td>( A' )</td>
<td>( A'' )</td>
<td>( A''' )</td>
</tr>
<tr>
<td>( B )</td>
<td>( B' )</td>
<td>( B'' )</td>
<td>( B''' )</td>
</tr>
<tr>
<td>( C )</td>
<td>( C' )</td>
<td>( C'' )</td>
<td>( C''' )</td>
</tr>
</tbody>
</table>

12. What observations/conjectures can you make about dilations and ordered pairs based on your work so far?

13. Complete this conjecture based on Table 1: The image of \( (a, b) \) after a dilation with center \( O \) and scale factor \( k \) has coordinates (__________, __________).
## Dilations — Part II

7. Record your observations about the characteristics of the triangles \( \triangle ABC \) and \( \triangle A'B'C' \) from \( \text{Dilations – Part I} \) in the table below.

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Original ( \triangle ABC )</th>
<th>( \triangle A'B'C' ) (Scale Factor of 2)</th>
<th>Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coordinates</td>
<td>((3, 5)) ((3, 2)) ((7, 2))</td>
<td>((3, 5)) ((3, 2)) ((7, 2))</td>
<td></td>
</tr>
<tr>
<td>Angle Measures</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Length of Sides</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Perimeter</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Area</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

8. Graph Quadrilateral \( JKLM \) on the back of your grid paper (from \textbf{Part I}). Dilate quadrilateral \( JKLM \) with a scale factor of 3 on your grid paper. Label the image \( J'K'L'M' \) and complete the table below.

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Original ( JKLM )</th>
<th>Quad. ( J'K'L'M' ) (Scale Factor of 3)</th>
<th>Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coordinates</td>
<td>((-2, 1)) ((-2, 3)), ((3, 3)), ((3, 1))</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Angle Measures</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Length of Sides</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Perimeter</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Area</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

9. Graph Quadrilateral \( QRST \) on the same grid paper. Dilate quadrilateral \( QRST \) with a scale factor of \( \frac{1}{2} \) on your grid paper. Label the image \( Q'R'S'T' \) and complete the table below.

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Original ( QRST )</th>
<th>Quad. ( Q'R'S'T' ) (Scale Factor of ( \frac{1}{2} ))</th>
<th>Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coordinates</td>
<td>((-2,-2)) ((-2, -4)), ((-6, -4)), ((-6, 2))</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Angle Measures</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Length of Sides</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Perimeter</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Area</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Dilations Investigation—Student Activity

10. A diameter of a circle has endpoints A and B. Graph the circle on your grid paper. Dilate the circle with a scale factor of 2. Label the coordinates of the image A' and B' and complete the table below.

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Original circle with diameter ( \overline{AB} )</th>
<th>New circle with diameter ( \overline{A'B'} )</th>
<th>Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coordinates</td>
<td>( A(4,1), B(4,-3) )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Length of the diameter</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Length of the radius</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Circumference</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Area</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

11. Based on the information in all four tables, what conjectures can you make about:

   e. Scale Factor used in the dilation and angle measures

   f. Scale Factor used in the dilation and side lengths/diameter

   g. Scale Factor used in the dilation and perimeter/circumference

   h. Scale Factor used in the dilation and area

12. Use the graph below to answer the following dilation questions.

   a) Find the coordinates of triangle \( A'B'C' \) after a dilation with a scale factor of 3.

      How does this change angle measures?

   b) Find the coordinates of triangle \( A'B'C' \) after a dilation with a scale factor of \( \frac{3}{2} \).

      How does this change the lengths of the sides?

   c) If the coordinates of \( A' \) are (-1.5, 1) after a dilation, find the coordinates of \( B' \) and \( C' \)?

      How does this change the perimeter of the triangle?

   d) If the coordinates of \( B' \) are (18, -12) after a dilation, find the coordinates of \( A' \) and \( C' \)?

      How does this change the area of the triangle?
Dilations Investigation—Student Activity

Name _______________________________________________________________

Dilations

Geometry
Dilations Investigation—Student Activity
Dilations Investigation—Homework
Geometry
Dilation WS
Name: ____________________
Date: ____________ Per.: _____

Dilate the following with the given scale factors.

1) scale factor = 2

2) scale factor = 3

3) scale factor = 2

4) scale factor = \( \frac{3}{4} \)

5) scale factor = \( \frac{3}{2} \)

6) scale factor = \( \frac{2}{3} \)

Dilate the following triangles with the given scale factors.

7) \( S_2 \)

Dilate the following with the given scale factors.

1) scale factor = 2

2) scale factor = 3

3) scale factor = 2

4) scale factor = \( \frac{3}{4} \)

5) scale factor = \( \frac{3}{2} \)

6) scale factor = \( \frac{2}{3} \)

Dilate the following triangles with the given scale factors.

7) \( S_2 \)
Dilations Investigation—Homework

8) $S_3$

9) $S_{3/2}$

10) $S_{1/2}$
Dilations Investigation—Homework

11) $S_2$

12) $S_{\frac{2}{3}}$

13) $S_{-3}$
Dilations Investigation—Homework

14) Use a ruler to find the scale factor of the following dilation. 

![Diagram of dilations]

a) How does the dilation change the angles measures? 

b) How does the dilation change the length of the sides? 

c) How does the dilation change the perimeter? 

d) How does the dilation change the area? 

15) The graph shows \( \triangle PQR \) with vertices \( P (2, 4) \), \( Q (8, 6) \), and \( R (6, 2) \) and \( \overline{SU} \) with endpoints \( S (5, 10) \) and \( U (15, 5) \).

At what coordinates would vertex \( T \) be placed to create \( \triangle STU \), a triangle that is a dilation of \( \triangle PQR \)?

- \( F \) \((12, 9)\)
- \( G \) \((16, 12)\)
- \( H \) \((20, 15)\)
- \( J \) \((24, 18)\)